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ON THE COVERING RADIUS OF SOME CODES OVER R = $Z_2 + uZ_2$, WHERE $u^2 = 0$

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ABSTRACT

In this correspondence, we give lower and upper bounds on the covering radius of codes over the ring R = $Z_2 + uZ_2$ where $u^2 = 0$ with respect to different distance. We also determine the covering radius of various Repetition codes, Simplex codes (Type α and Type β) and their dual and give bounds on the covering radius for MacDonald codes of both types over R.

KEYWORDS: Covering Radius, Codes over Finite Rings, Simplex Codes, Hamming Codes

1 INTRODUCTION

In the last decade, there are many researchers doing research on code over finite rings. In particular, codes over Z_4, $Z_2 + uZ_2$ where $u^2 = 0$ received much attention [1, 2, 3, 4, 5, 9, 11, 12, 14, 16, 17]. The covering radius of binary linear codes were studied [6, 7]. Recently the covering radius of codes over Z_4 has been investigated with respect to Lee and Euclidean distances [1, 15]. In 1999, Sole et al gave many upper and lower bounds on the covering radius of a code over Z_4 with different distances. In the recent paper [15], the covering radius of some particular codes over Z_4 have been investigated. In this correspondence, we consider the ring R = $Z_2 + uZ_2$ where $u^2 = 0$. In this paper, we investigate the covering radius of the Simplex codes (both types) and their duals, MacDonald codes and repetition codes over R. We also generalized some of the known bounds in [1]. A linear code C of length n over R is an additive subgroup of R^n. An element of C is called a codeword of C and a generator matrix of C is a matrix whose rows generate C. The Hamming weight $w_H(x)$ of a vector x in R^n is the number of non-zero components. The Lee weight for a codeword $x = (x_1, x_2, \ldots, x_n)$ is defined by

$$w_L(x) = \sum_{i=1}^{n} w_L(x_i), \text{ where } w_L(x_i) = \begin{cases} 0 & if \quad x_i = 0 \\ 1 & if \quad x_i = 1 \, or \, 1+u \\ 2 & if \quad x_i = u \end{cases}$$

The Lee distance between the codewords x and y $\in R^n$ is defined as $d_L(x, y) = w_L(x - y)$. The Euclidean weight is given by the relation

$$w_E(x) = \sum_{i=1}^{n} w_E(x_i), \text{ where } w_E(x_i) = \begin{cases} 0 & if \quad x_i = 0 \\ 1 & if \quad x_i = 1 or 1+u \\ 4 & if \quad x_i = u \end{cases}$$

The Euclidean distance between the codewords x and y $\in R^n$ is defined as $d_E(x, y) = w_E(x - y)$.

A linear Gray map φ from $R \to Z_2^2$ is defined by φ(x + uy) = (y, x + y), for all x + uy ∈ R. The image φ(C), of a linear code C over R of length n by the Gray map, is a binary code of length 2n with same cardinality [16]. The dual code $C^\perp$ of C is defined as $\{x \in R^n \mid x.y = 0 \text{ for all } y \in C\}$ where $x.y = \sum_{i=0}^{n} x_i y_i \pmod 2$. C is self-orthogonal if $C \subseteq C^\perp$ and C is self-dual if $C = C^\perp$. Two codes are said to be equivalent if one can be obtained from the other by permuting the coordinates or changing the signs of certain coordinates or multiplying non-zero element in a fixed column. Codes differing by only a permutation of coordinates are called permutation-equivalent.

Any linear code C over R is equivalent to a code with generator matrix G of the form

$$G = \begin{bmatrix} I_{k_0} & A & B \\ 0 & uI_{k_1} & uD \end{bmatrix}, \tag{1.1}$$

where A, B and D are matrices over R. Then the code C contain all codewords $[v_0, v_1]G$, where v_0 is a vector of length k_1 over R and v_1 is a vector of length k_2 over Z_2. Thus C contains a total of $4^{k_1}2^{k_2}$ codewords. The parameters of C are given [n, $4^{k_1}2^{k_2}$, d] where d represents the minimum distance of C. In[11], we associate to the code C, two binary codes. The residue code C_1 is defined as $C_1 = \{x \in Z_2^n \mid \exists\ y \in Z_2^n$ and $x + uy \in C\}$ and the torsion code $C_2 = \{x \in Z_2^n \mid ux \in C\}$. A vector v is a 2-linear combination of the vectors $v_1, v_2, \ldots, v_k$ if $v = l_1 v_1 + \ldots + l_k v_k$ with $l_i \in Z_2$ for $1 \le i \le k$. A subset $S = \{v_1, v_2, \ldots, v_k\}$ of C is called a 2-basis for C if for each i = 1, 2, ..., k −1, 2vi is a 2-linear combination of $v_{i+1}, \ldots, v_k$, $2v_k = 0$, C is the 2-linear span of S and S is 2-linearly independent [18]. The number of elements in a 2-basis for C is the 2-dimension of C. It is easy to verify that the rows of the matrix

$$B = \begin{bmatrix} I_{K_0} & A & B \\ uI_{k_0} & uA & uB \\ 0 & uI_{k_1} & uD \end{bmatrix} \tag{1.2}$$

form a 2-basis for the code C generated by G given in (1.1). A linear code C over R (over Z_2) of length n, 2-dimension k, minimum distance d_H and d_L is called an $[n, k, d_H, d_L]$ ($[n, k, d_H]$) or simply an [n, k] code. In this paper, we define the covering radius of codes over R with respect to different distances and in particular study the covering radius of Simplex codes of type α and β namely, S_k^{α} and S_k^{β} and their duals, MacDonald codes and repetition codes over R. Section 2 contains basic results for the covering radius of codes over R. Section 3 determines the covering radius of different types of repetition codes. Section 4 determines the covering radius of Simplex codes and its dual and finally section 5 determines the bounds on the covering radius of MacDonald codes.

2 COVERING RADIUS OF CODES

Let d be the general distance out of various possible distances (such as Hamming, Lee, and Euclidean). The covering radius of a code C over R with respect to a general distance d is given by $r_d(C) = \max_{u \in R^n}\left\{\min_{c \in C}\{d(c,u)\}\right\}$.

It is easy to see that $r_d(C)$ is the least positive integer r_d such that $R^n = \bigcup_{c \in C} S_{r_d}(c)$ where $S_{r_d}(u) = \{v \in R^n \mid d(u,v) \le r_d\}$ for any $u \in R^n$. The translate $u+C = \{u + c \mid c \in C\}$ is called the coset of C where $u \in R^n$. A vector of minimum weight in a coset is called a coset leader. The following propositions are straight forward generalization from [1].

Proposition 2.1

The covering radius of C with respect to the general distance d is the largest minimum weight among all cosets.

Proposition 2.2

Let C be a code over R and φ(C) the generalized Gray map image of C. Then $r_L(C) = r_H(\varphi(C))$.

Now, we give several lower and upper bounds on the covering radius of codes over R with respect to general weight. The proof of Proposition 2.3 and Theorem 2.4 being similar to the case of Z_4 [1], is omitted.

Proposition 2.3 (Sphere-Covering Bounds)

For any code C of length n over R,

$$\frac{2^{2^{s-1}n}}{|C|} \leq \sum_{i=0}^{r_d(C)} \binom{2^{s-1}n}{i}$$

We consider the two upper bounds on the covering radius of a code over R with respect to general weight. Let C be a code over R and let $s(C^{\perp}) = |\{\, i \mid A_i(C^{\perp}) \neq 0, i \neq 0 \,\}|$ where $A_i(C^{\perp})$ is the number of codewords of various possible distances i in $C^{\perp}$.

Theorem 2.4 (Delsarte Bound)

Let C be a code over R, then $r_d(C) \leq s(C^{\perp})$.

The following result of Mattson [6] is useful for computing covering radius of codes over rings generalized easily from codes over finite fields.

Proposition 2.5 (Mattson)

If C_0 and C_1 are codes over R generated by matrices G_0 and G_1 respectively and if C is the code generated by $G = \left(\begin{array}{c|c} 0 & G_1 \\ \hline G_0 & A \end{array}\right)$ then $r_d(C) \leq r_d(C_0) + r_d(C_1)$ and the covering radius of D (concatenation of C_0 and C_1) satisfy the following $r_d(D) \geq r_d(C_0) + r_d(C_1)$, for all distances d over R. Since the covering radius of C generated by G = [A|B] is greater than or equal to $r_d(C_A) + r_d(C_B)$ where C_A and C_B are codes generated by A and B respectively, this implies rd (D) $\geq r_d(C_0) + r_d(C_1)$ because C_1 is a subcode of the code generated $[G_1|A]$.

3 COVERING RADIUS OF REPETITION CODES

A q-ary repetition code C over a finite field $F_q = \{\alpha_0 = 0, \alpha_1 = 1, \alpha_2, \alpha_3, \ldots, \alpha_{q-1}\}$ is an [n, 1, n] code $C = \{\bar{\alpha} \mid \alpha \in F_q\}$, where $\bar{\alpha} = (\alpha, \alpha, \cdots, \alpha)$. The covering radius of C is $\left\lceil \frac{n(q-1)}{q} \right\rceil$ [13]. Using this, it can be seen easily that the covering radius of block (of size n) repetition code [n(q-1),1,n(q-1)] generated by $G = [\overbrace{11\cdots1}^{n}\overbrace{\alpha_2\alpha_2\cdots\alpha_2}^{n}\overbrace{\alpha_3\alpha_3\cdots\alpha_3}^{n}\cdots\overbrace{\alpha_{q-1}\alpha_{q-1}\cdots\alpha_{q-1}}^{n}]$ is $\left\lceil \frac{n(q-1)^2}{q} \right\rceil$ since it will be equivalent to a repetition code of length (q − 1)n. Consider the repetition code over R. There are two types of them of length n viz. unit repetition code C_β: [n, 2, n, n] generated by G_β = [11 . . . 1] and zero divisor repetition code C_α : [n, 1, n, 2n] generated by G_α = [uu . . . u]. With respect to the Hamming distance the covering radius of C_β is $\left\lceil \frac{n(q-1)}{q} \right\rceil$ but covering radius of C_α is n.

The following result determines the covering radius with respect Lee distance and Euclidean distance.

Theorem 3.1

$r_L(C_\alpha) = n$, $r_E(C_\alpha) = 2n$, $r_L(C_\beta) = n$ and $r_E(C_\beta) = \frac{3n}{2}$.

Proof. Note that $\varphi(C_\alpha)$ is a binary repetition code of length 2n hence $r_L(C_\alpha) = \frac{2n}{2} = n$. Now by definition $r_E(C_\alpha) = \max_{u \in R^n}\{d_E(x, C_\alpha)\}$. Let $x = \overbrace{uu\cdots u}^{\frac{n}{2}}\overbrace{000\cdots 0}^{\frac{n}{2}} \in R^n$, then $d_E(x, 0) = d_E(x, u) = 2n$. Thus $r_E(C_\alpha) \geq 2n$. On the other hand if $x \in R^n$ has a composition $(\omega_0, \omega_1, \omega_2, \omega_3)$, where $\sum_{i=0}^{3} w_i = n$ then $d_E(x, \bar{0}) = n - \omega_0 + 3\omega_2$ and $d_E(x, \bar{u}) = n - \omega_2 + 3\omega_0$. Thus $d_E(x, C_\alpha) = \min\{n - \omega_0 + 3\omega_2, n - \omega_2 + 3\omega_0\} \leq n + \omega_0 + \omega_2 \leq n + n = 2n$. Hence $r_E(C_\alpha) = 2n$. Similar arguments can be used to show that $r_E(C_\beta) \leq \frac{3n}{2}$. To show that $r_E(C_\beta) \geq \frac{3n}{2}$, let $x = \overbrace{000\cdots 0}^{t}\overbrace{111\cdots 1}^{t}\overbrace{uu\cdots u}^{t}\cdots\overbrace{1+u1+u\cdots 1+u}^{n-3t} \in R^n$, where $t = \left\lfloor \frac{n}{4} \right\rfloor$, then $d_E(x, \bar{0}) = n+2t$, $d_E(x, \bar{1}) = 4n-10t$, $d_E(x, \bar{u}) = n+2t$ and $d_E(x, \overline{1+u}) = 6t$. Thus $r_E(C_\beta) \geq \min\{4n - 10t, n + 2t, 6t\} \geq \frac{3n}{2}$. The proof of $r_L(C_\beta) = n$ is simple so we omit it.

In order to determine the covering radius of Simplex and MacDonald codes over R, we need to define few block repetition codes over R and find their covering radius. To determine the covering radius of R block (three blocks each of size n) repetition code $BRep_\alpha^{3n}$: [3n, 2, 2n, 4n, 6n] generated by $G = [\ \overbrace{111\cdots 1}^{n}\overbrace{uu\cdots u}^{n}\cdots\overbrace{1+u1+u\cdots 1+u}^{n}\ ]$ note that the code has constant Lee weight 4n. Thus for $x = 11\cdots 1 \in R^{3n}$, we have $d_L(x, BRep_\alpha^{3n}) = 3n$. Hence by definition, $r_L(BRep_\alpha^{3n}) \geq 3n$. On the other hand, its Gray image $\varphi(BRep_\alpha^{3n})$ is equivalent to binary linear code [6n, 2, 4n] with the generator matrix $\begin{pmatrix} \overbrace{11\cdots 1}^{2n} & \overbrace{11\cdots 1}^{2n} & \overbrace{00\cdots 0}^{2n} \\ \underbrace{11\cdots 1}_{2n} & 00\cdots 0 & 11\cdots 1 \end{pmatrix}$. Thus the covering radius $r_L(BRep_\alpha^{3n}) \leq \frac{4n}{2} + \frac{2n}{2} = 3n$. Thus, we have $r_L(BRep_\alpha^{3n}) = 3n$. With respect Euclidean distance, it is clear that $r_E(BRep_\alpha^{3n}) \geq \frac{3n}{2} + 2n + \frac{3n}{2} = 5n$. Let $x = (u|v|w) \in R^{3n}$ with u, v and w have compositions (r_0, r_1, r_2, r_3), (s_0, s_1, s_2, s_3) and (t_0, t_1, t_2, t_3) respectively such that sum of each component composition is n, then $d_E(x, \bar{0}) = 3n - r_0 + 3r_2 - s_0 + 3s_2 - t_0 + 3t_3$, $d_E(x, c_1) = 3n - r_1 + 3r_3 - s_2 + 3s_0 - t_3 + 3t_1$, $d_E(x, c_2) = 3n - r_2 + 3r_0 - s_0 + 3s_2 - t_2 + 3t_0$ and $d_E(x, c_3) = 3n - r_3 + 3r_1 - s_2 + 3s_0 - t_1 + 3t_3$. Thus $d_E(x, BRep_\alpha^{3n}) \leq 3n + \min\{3r_2 + 3s_2 + 3t_2 - r_0 - s_0 - t_0, 3r_3 + 3s_0 + 3t_1 - r_1 - s_2 - t_3, 3r_0 + 3s_2 + 3t_0 - r_2 - s_0 - t_2, 3r_1 + 3s_0 + 3t_3 - r_3 - s_2 - t_1\} \leq 3n + \frac{1}{2}\{2n + 2s_0 + 2s_2\} \leq 5n$. Thus we have the following theorem.

Theorem 3.2

$r_L(BRep_\alpha^{3n}) = 3n$ and $r_E(BRep_\alpha^{3n}) = 5n$.

One can also define a R block (two blocks each of size n) repetition code Brep $_{\alpha}^{2n}$: [2n, 2, n, 2n, 4n] generated by $G = \left[\overbrace{11 \cdots 1}^{n} \overbrace{uu \cdots u}^{n} \right]$. We have following theorem (its proof is similar to the proof of Theorem 3.2) so we omit it.

Theorem 3.3

$r_L(\text{BRep}_{\alpha}^{2n}) = 2n$ and $r_E(\text{BRep}_{\alpha}^{2n}) = \frac{7n}{2}$

Block code Brep $_{\alpha}^{2n}$ can be generalized to a block repetition code (two blocks of size m and n respectively)

BRep^{m+n} : [m + n, 2, m, min{2m, m + 2n}, min{4m, m + 4n}] generated by $G = \left[\overbrace{11 \cdots 1}^{m} \overbrace{uu \cdots u}^{n} \right]$. Theorem 3.3 can be easily generalized using similar arguments to the following.

Theorem 3.4

$r_L(\text{BRep}^{m+n}) = m + n$ and $r_E(\text{BRep}^{m+n}) = 2n + \frac{3m}{2}$

4 SIMPLEX CODES OF TYPE α AND β OVER *R*

Quaternary simplex codes of type α and β have been recently studied in [2]. Type α Simplex code S_k^{α} is a linear code over *R* with parameters $[2^{2k}, 2k, 2^{2k-1}, 2^{2k}, 3.2^{2k-1}]$ and an inductive generator matrix given by

$$G_k^{\alpha} = \left[\begin{array}{c|c|c|c} 00\cdots0 & 11\cdots1 & uu\cdots u & 1+u1+u\cdots1+u \\ \hline G_{k-1}^{\alpha} & G_{k-1}^{\alpha} & G_{k-1}^{\alpha} & G_{k-1}^{\alpha} \end{array} \right] \tag{4.1}$$

with G_1^{α} =[0 1 *u* 1+*u*]. The dual code of S_k^{α} is a $[2^{2k}, 2^{2k+1} - 2k]$ code. Type simplex code S_k^{β} is a punctured version of S_k^{α} with parameters $[2^{k-1}(2^k - 1), 2k, 2^{2(k-1)}, 2^{(k-1)}(2^k - 1), 2^k(3.2^{k-2} - 1)]$ and an inductive generator matrix given by

$$G_2^{\beta} = \left[\begin{array}{cccc|c|c} 1 & 1 & 1 & 1 & 0 & u \\ \hline 1 & 1 & u & 1+u & 1 & 1 \end{array} \right] \tag{4.2}$$

and for $k > 2$ $$G_k^{\beta} = \left[\begin{array}{c|c|c} 11 \cdots 1 & 00 \cdots 0 & uu \cdots u \\ \hline G_{k-1}^{\alpha} & G_{k-1}^{\beta} & G_{k-1}^{\beta} \end{array} \right] \tag{4.3}$$

where G_{k-1}^{α} is the generator matrix of S_{k-1}^{α}. For details the reader is refereed to [2]. The dual code of S_k^{β} is a $[2^{k-1}(2^k - 1), 2^{2k} - 2^k - 2k]$ type α code with minimum Lee weight $d_L = 3$.

Theorem 4.1

$2^{2k} \le r_L(S_k^{\alpha}) \le 2^{2k}+1$ *and* $r_E(S_k^{\alpha}) \le \frac{5.4^k + 5}{3}$

Proof. Let $x = 11\ldots.. 1 \in R^n$, we have $d_L(x, S_k^{\alpha}) = 2^{2k}$. By definition, $r_L(S_k^{\alpha}) \geq 2^{2k}$. To find the upper bound, by equation 4.1, the result of Mattson for finite rings and using Theorem 3.2, we get

$$r_L(S_k^{\alpha}) \leq r_L(S_{k-1}^{\alpha}) + r_L(<\overbrace{11\cdots1}^{2^{2(k-1)}}\ \overbrace{uu\cdots u}^{2^{2(k-1)}}\ \overbrace{1+u1+u\cdots1+u}^{2^{2(k-1)}}>)$$

$$= r_L(S_{k-1}^{\alpha}) + 3.2^{2(k-1)}$$

$$= 3.2^{2(k-1)} + 3.2^{2(k-2)} + 3.2^{2(k-3)} + \ldots.. + 3.2^{2.2} + 3.2^{2.1} + r_L(S_1^{\alpha})$$

$r_L(S_k^{\alpha}) \leq 2^{2k}+1$(since $r_L(S_1^{\alpha}) = 5$). Thus $2^{2k} \leq r_L(S_k^{\alpha}) \leq 2^{2k}+1$.

Similar arguments can be used to show $r_E(S_k^{\beta}) \leq 5(4^{(k-1)} + 4^{(k-2)} + 4^{(k-3)} + \ldots. + 4^1 + 1) - \frac{11}{2} + r_E(S_1^{\alpha})$.

$r_E(S_k^{\alpha}) \leq \frac{5.4^k+5}{3}$ (since $r_E(S_1^{\alpha}) = 8$). Similar arguments will compute the covering radius of Simplex codes of type β. We provide an outline of the proof.

Theorem 4.2

$r_L(S_k^{\beta}) \leq 2^{k-1}(2^k - 1) - 1$ *and* $r_E(S_k^{\beta}) \leq \frac{5.4^k - 6.2^k - 8}{6}$

Proof. By equation 4.3, Proposition 2.5 and Theorem 3.4, we get

$$r_L(S_k^{\beta}) \leq r_L(S_{k-1}^{\beta}) + r_L(<\overbrace{11\cdots1}^{4^{(k-1)}}\ \overbrace{uu\cdots u}^{2^{(2k-3)}-2^{(k-2)}}>)$$

$$= r_L(S_{k-1}^{\beta}) + 2^{(2k-2)} + 2^{(2k-3)} - 2^{(k-2)}$$

$$\leq (2^{(2k-2)} + 2^{(2k-4)} + \ldots + 2^4) + (2^{(2k-3)} + 2^{(2k-5)} + \ldots + 2^3) - (2^{(k-2)} + 2^{(k-3)} + \ldots + 2) + r_L(S_2^{\beta})$$

Thus $r_L(S_k^{\beta}) \leq 2^{k-1}(2^k - 1) - 1$(since $r_L(S_2^{\beta}) = 5$). Similarly, by using Theorem 3.4, we derive

$$r_E(S_k^{\beta}) \leq \frac{3}{2}(4^{(k-1)} + 4^{(k-2)} + \ldots + 4^2) + (4^{(k-1)} + 4^{(k-2)} + \ldots + 4^2) - 2(2^{k-2} + 2^{k-3} + \ldots + 2) + r_E(S_2^{\beta})$$

$r_E(S_k^{\beta}) \leq \frac{5.4^k - 6.2^k - 8}{6}$ (since $r_E(S_2^{\beta}) = 8$).

Theorem 4.3

$r_L(S_k^{\alpha\perp}) = 1$, $r_L(S_k^{\beta\perp}) = 2$, $r_E(S_k^{\alpha\perp}) \leq 4$ and $r_E(S_k^{\beta\perp}) \leq 4$.

Proof. By Delsarte Bound, $r_L(S_k^{\alpha\perp}) \le 1$ and $r_L(S_k^{\beta\perp}) \le 2$. Thus equality follows in the first case. For second case, note that $r_L(S_k^{\beta\perp}) \neq 1$, by sphere covering bound. The result for Euclidean distance follows from Delsarte bound.

5 MACDONALD CODES CODES OF TYPE α AND β OVER R

The q-ary MacDonald code $M_{k,t}(q)$ over the finite field F_q is a unique $\left[\frac{q^k - q^t}{q-1}, k, q^{k-1} - q^{t-1}\right]$ code in which every nonzero codeword has weight either q^{k-1} or $q^{k-1} - q^{t-1}$ [10]. In [8], authors have defined the MacDonald codes over R using the generator matrices of simplex codes. For $1 \le t \le k-1$, let $G_{k,t}^{\alpha}$ $(G_{k,t}^{\beta})$ be the matrix obtained from $G_{k,}^{\alpha}$ $(G_{k,}^{\beta})$ by deleting columns corresponding to the columns of

$$G_t^{\alpha} \quad (G_t^{\beta}) \text{ . i. e, } G_{k,t}^{\alpha} = \left[G_k^{\alpha} \setminus \frac{0}{G_t^{\alpha}} \right] \tag{5.1}$$

$$\text{and } G_{k,t}^{\beta} = \left[G_k^{\beta} \setminus \frac{0}{G_t^{\beta}} \right] \tag{5.2}$$

where $[A \setminus B]$ denotes the matrix obtained from the matrix A by deleting the columns of the matrix B and **0** in 5.1 (respectively(5.2)) is a $(k - t) \times 2^{2t}$ (respectively $(k - t) \times 2^{t-1}(2^t - 1)$) zero matrix. The code $M_{k,t}^{\alpha} : [2^{2k} - 2^{2t}, 2k]$ $(M_{k,t}^{\beta} : [(2^{k-1} - 2^{t-1})(2^k + 2^t - 1), 2k])$ generated by the matrix $G_{k,t}^{\alpha}$ $(G_{k,t}^{\beta})$ is the punctured code of S_k^{α} (S_k^{β}) and is called a *MacDonald code* of type α **(β).** Next Theorem provides basic bounds on the covering radius of MacDonald codes.

Theorem 5.1

$$r_L(M_{k,t}^{\alpha}) \le 4^k - 4^r + r_L(M_{r,t}^{\alpha}) \text{ for } t < r \le k,$$

$$r_E(M_{k,t}^{\alpha}) \le \frac{5}{3}(4^k - 4^r) + r_E(M_{r,t}^{\alpha}) \text{ for } t < r \le k.$$

Proof. By Theorem 3.2,

$$\begin{aligned} r_L(M_{k,t}^{\alpha}) &\le 3.2^{(2k-2)} + r_L(M_{k-1,t}^{\alpha}) \\ &\le 3.2^{(2k-2)} + 3.2^{(2k-4)} + \dots + 3.2^{2r} + r_L(M_{r,t}^{\alpha}), \ k \ge r > t. \\ &= 4^k - 4^r + r_L(M_{r,t}^{\alpha}). \end{aligned}$$

Similar arguments holds for $r_E(M_{k,t}^{\alpha})$. Similarly using equation 5.2, Proposition 2.5 and Theorem 3.4, following bounds can be obtained for type β MacDonald code.

Theorem 5.2

$$r_L(M_{k,t}^{\beta}) \le 2^{(k-1)}(2^k - 1) + 2^{(r-1)}(1 - 2^r) + r_L(M_{r,t}^{\beta}) \text{ for } t < r \le k,$$

$$r_E(M^{\beta}_{k,t}) \le \frac{2^k(5.2^k - 6) + 2^r(6 - 5.2^r)}{6} + r_E(M^{\beta}_{r,t}) \text{ for } t< r \le k$$

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